Rayleigh - Bénard Convection

Arunn Narasimhan [arunn@engr.smu.edu **]**

Laboratory for Porous Materials Applications
Mechanical Engineering Department
Southern Methodist University
Dallas, TX - 75275 0337, USA

Last update : 07/07/01

THE NATURE OF THE PROBLEM

The most fascinating thing about convection is that even the simplest system undergoing convective motion cannot yet be given an exact analytical mathematical description. The partial differential equation(s) that describe the convective flow analytically have been studied for the past 200 years- with rewarding results - but the exact analytical solutions of these are yet to be found ! The nature of the theoretical difficulties can be understood well if we realize that even a simple system undergoing convective energy interaction requires a complete knowledge of the Fluid Mechanics and Heat Transfer involved in the process.

A fluid layer heated from below, a supposedly simple system of convective interaction, experiences forces, that drive the convective flow, resulting from the buoyancy of the heated layer the magnitude of such forces depending on the temperature difference prevailing between the top and bottom portion of the fluid layer. The complexity is enhanced further by the fact that the temperature distribution is affected to a large extent by the convective flow itself, which carries heat from the bottom to the relatively colder top portion of the fluid. In other words the driving force which causes the flow itself is driven to modifications by the flow !

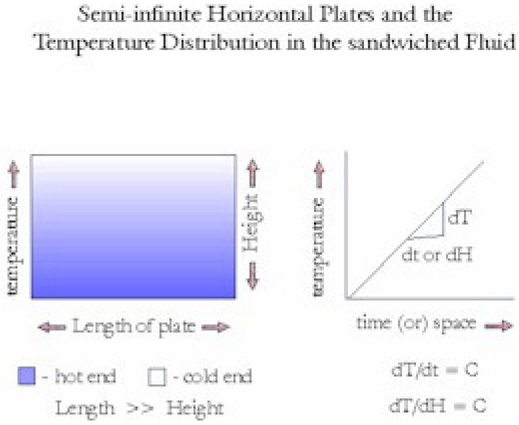
EARLY THEORIES

Earliest description of convection was written in the 1790s by **Benjamin Thompson, Count Rumford** - which he used to account for the transfer of heat in an apple pie ! Only in the 1900s were the systematic investigations undertaken. The most significant and pertinent experimental work was carried out by the Frenchman **Henri Benard**. He studied a seemingly simple convective system which he never knew was so complicated that the real physics behind it was uncovered only recently ! In the 1900s convection was taken as one of the myriad things that **John William Strutt, Lord Rayleigh** studied in his illustrious and prolific carrier.

Out of the more competent Physicists some have had the insight of originating a new field of study thereby making the first word(s) about it. But Lord Rayleigh, according to Chandrashekar, had said the last words of many subjects and sealed them once and for all ! In one of his last articles, published in 1916, he attempted to explain what is now known as **Rayleigh-Benard Convection**. Though his explanation was superceded in later years, his work remains as the starting point for most of the modern theories of convection.

RAYLEIGH'S THEORY

MODEL



A fluid with simplified properties, as against a real one, is considered in the two dimensional model to be constructed to explain convection. A thin layer of the fluid is confined fully between two semi-infinite (small, definite thickness vertically and long, infinite length horizontally as shown in the adjacent figure) flat plates so that there is no gap (free surface). By a thin layer here we again mean that the horizontal dimension of the fluid layer is very large when compared to that of the vertical.

This ensures the interference of the side boundaries (walls etc.) to a minimum. The fluid has to be heated from the bottom in such a way that the temperature of the bottom portion of the fluid remains uniform (spatial invariance) and steady (temporal invariance). Similarly it is assumed that the top portion also behaves in the same fashion so that the temperature gradient across the height also is uniform. This, in other words, means that the graph of Temperature Vs Time is a straight line(as shown).

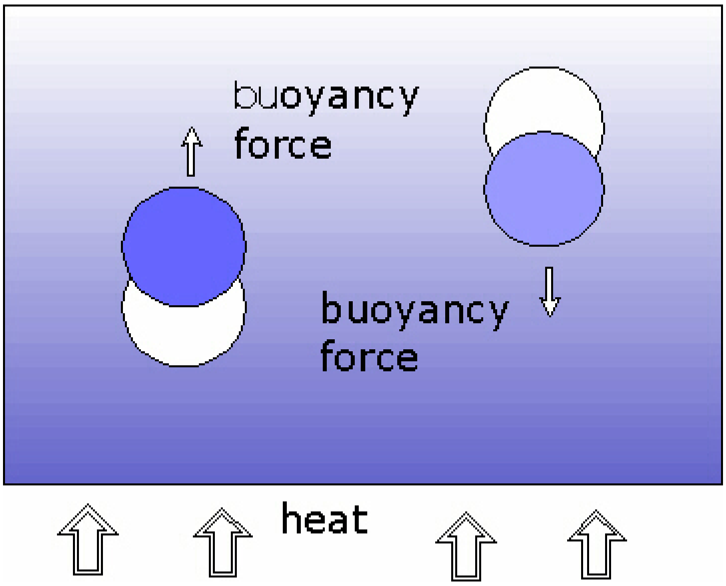
ASSUMPTIONS

The fluid itself is assumed to be

1. Incompressible - which is valid as the layer is of very small dimensions;
2. The density of the fluid is the only property that gets affected by the change in the temperature across it;
3. It experiences uniform Gravitational force on the entire volume.

In this model consider a packet of fluid of random dimension whose displacement above or below from its present position is controlled by forces whose nature is yet to be seen but are nevertheless responsible for convection. The study of the forces which affect the fluid packet's motion will lead us to a better understanding of the mechanism involved. The packet considered can be of any size and shape but the displacement must be small. The initial displacement of the packet need not be due to any imbalance in the forces we are studying but a random displacement from the mean position - which will eventually occur if we wait long enough - is sufficient. For understanding what is happening in the model one has to be familiar with some basic concepts : **buoyancy**, **viscosity**, **surface tension** and **thermal diffusivity** of fluids.

BUOYANCY



CASE1 : Without Heating buoyancy effects will be due to the pressure difference across the fluid packet which when balanced by the weight force of the packet, ensures static equilibrium.

CASE 2 : With Heating the fluid packet considered has lesser density at the bottom relative to the surrounding which makes it rise owing to the increase in buoyancy force which disrupts the static equilibrium.

If in the figure, in the fluid packet considered, water is filled and allowed to get immersed to a certain depth as shown, it will stay in the position inside the trough maintaining its static equilibrium. This is because the weight of the packet is balanced by the upward reaction force, by the water in the trough, called the buoyancy force. The pressure in the water trough increases as we go down because the weight of the water layer above each point also contributes to the net force experienced by that point. So this static pressure is greater on the bottom side of the packet than the upper. This balance in forces can get affected by the actual weight of the fluid in the packet and the pressure difference across the packet. For a heavier packet the weight force increases causing the packet to sink to a different height where the upward buoyancy force equals the weight force to make the packet float. So heavier, more massive, objects of identical volume hence with higher density, sink when compared to lighter objects.

EXPLANATION

Consider the same fluid packet at the bottom of the trough in the earlier figure with heat supplied to it from below, as shown. This packet considered has a higher temperature and so has lesser density when compared to the average density of the entire layer. A similar packet at the top side will have relatively higher density due to its lesser temperature. In the bottom side, as long as the fluid packet remains in its position it is surrounded by fluid of identical average density and so maintains its static equilibrium with the surrounding. Suppose now due to some random fluctuation a displacement is given to the packet in the upward direction. This will result in an imbalance in the forces acting on the packet.

The packet which is originally of lesser density than the surrounding average density due to its higher temperature now is pushed up into a region of higher density. This creates a positive buoyancy as explained above which causes the packet to raise. The raise will be sustained till the density of the fluid packet while raising equals that of the surrounding. At this point it will simply float as the static equilibrium is restored. The upward force is proportional to the density difference and volume of the packet. As the fluid packet raises through regions of relatively colder fluid whose average density progressively increases due to the lack of additional heat, it results in an increased density gradient between the packet and the surrounding which accelerates the raise.

On similar analysis the downward push of a packet of fluid makes it enter a region of lesser average density resulting in the `heaviness' of the packet thus propelling it down. It would sink getting its initial disturbance enhanced. Thus the whole of the fluid layer is eventually overturned resulting in a circulation of the fluid between the hot and cold ends.

It seems from this analysis that convection will be observed in a fluid region whenever there is a temperature gradient, however small it may be. But such sensitive dependence of the initiation of the flow on the temperature gradient is not observed in actual circumstances. There seems to be a cut-off value, of some variable which governs the phenomenon, beyond which only convective flow results. This was, characteristically, explained by Lord Rayleigh.

RAYLEIGH NUMBER AND ITS PHYSICAL SIGNIFICANCE

The onset of convection has to take into consideration two more modes of energy dissipation in the fluid. In other words, the force imbalance equation which explains the convective motion has to be recast to accommodate two more forces. One of our initial assumptions is that before the temperature gradient prevails the fluid is at rest and is not subjected to any external influence which might induce motion. So when the fluid tries to move, or circulate, it does so with minimum velocity. When the fluid packet moves, its motion is impeded by the `viscous drag' between it and the surrounding fluid.

Viscosity, as we know, is internal fluid resistance offered to a change in the momentum. It is given by the formula

$$\mathbf{t = m\ du/dy}$$

where **t** = the shear stress applied **m** = the dynamic viscosity and **du / dy** = the change in the velocity component in a perpendicular direction. It is like a frictional force acting in the opposite direction of motion. In our fluid packet, this acts against the buoyancy force and tries to impede motion. If the magnitude of the viscous drag force equals the buoyancy force, motion will cease.

The second dissipative effect is from the fact that convection is not the only mode of heat transfer that could happen in the given circumstance - **Conduction** and **Radiation** being the other two. Out of this Radiative effects are predominant only at very large temperature values but Conductive transfer cannot be ignored. In a real non-adiabatic situation the fluid packet is displaced into a cooler surrounding due to the buoyancy force. This immediately causes the packet to diffuse out the heat energy to the surrounding fluid as a temperature difference prevails.

The fundamental definition of Heat deigns the molecules in the warm packet to have higher average velocity than that of the surrounding. This makes the molecules in the packet to jiggle more freely thereby exchanging energy with the surrounding molecules of lesser velocity resulting in the equalization of velocities. This results ultimately in the premature cooling of the packet than was originally envisaged. For the fluid packet coming down from a cooler environment the transfer is in the other way leading to similar results.

So if the local temperature difference is reduced by heat diffusion it results in a reduction in the buoyancy force. It is necessary that the buoyancy force, which is the result of the temperature gradient, must exceed the dissipative forces of viscous drag and heat diffusion to ensure the onset of convective flow. The Gravitational potential energy given out by the displacement of the fluid packet up and down must be more than the `*fluid brake*' and `heat diffusion'. To realize better the influence of these forces on the onset of convection, these forces are expressed as a non-dimensional number called the **Rayleigh Number** which is the buoyant force divided by the product of the viscous drag and the rate of heat diffusion.

$$\mathbf{Ra = gbDTL^3 / an}$$

where **b** is the coefficient of thermal expansion **DT** the temperature difference between hot and cold end L the width (vertical distance between the walls) **a** the thermal diffusivity and **n** the kinematic viscosity of the fluid. Convection sets in when the Rayleigh number exceeds a certain critical value. Thus Rayleigh Number is a quantitative measure/ representation of *when* the 'switch' from conductive to convective transport happens for a given configuration (henceforth, the *dominant* energy transport mechanism would be convection).

MODIFICATIONS OF RAYLEIGH THEORY

Lord Rayleigh's analysis of the problem of convective flow was initiated by the experiments of Benard. While trying to explain those experiments Rayleigh devised the theory explained above. This theory unfortunately assumes a model experiment which is in a subtle way different from the actual experiments of Benard. So this theory fails in explaining those experiments.

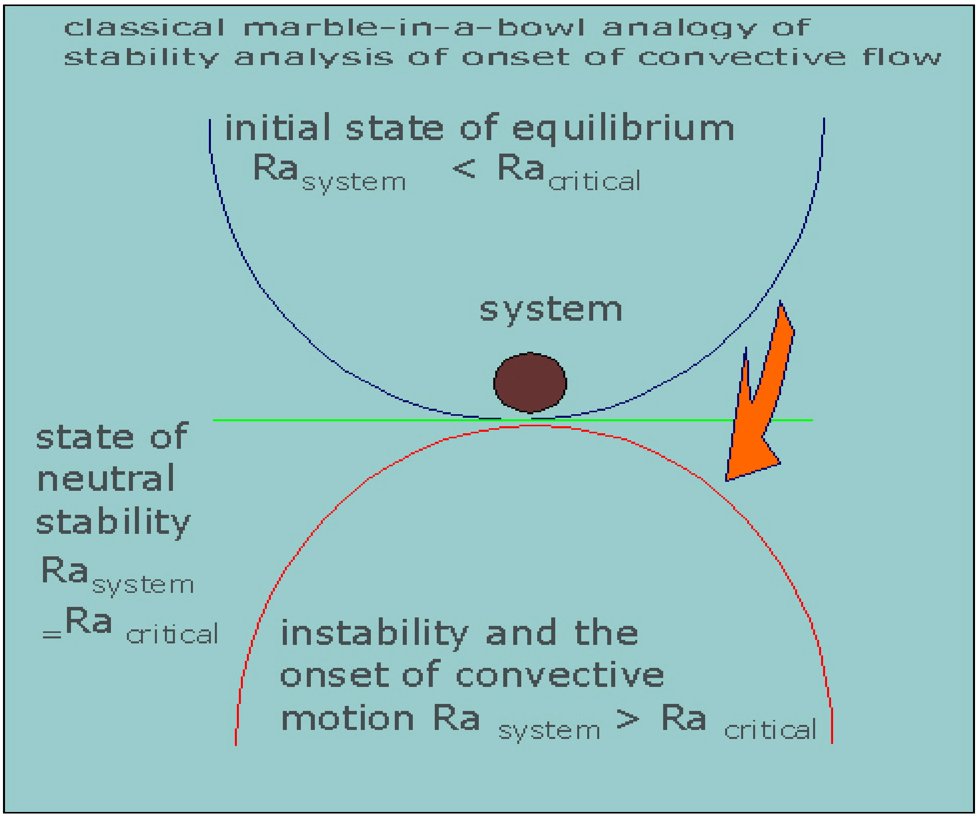
The experimental conditions Benard employed was different in the sense that the fluid layer is not fully confined between two horizontal rigid plates, as assumed in the model above, but is open to air in the upper surface. Since the surface is free, Surface Tension forces can affect the flow, which can dominate over even the buoyancy force.

The predictions of the Rayleigh Theory will be in error for the experiments and a successful alternate theory was introduced in 1958 by **J.R.A.Pearson** of the Imperial College of Science & Technology in London. A new dimensionless number called the **Marangoni Number**, named after the 19th century Italian investigator, which includes the effects of surface tension was introduced to explain the experiments of Benard.

Rayleigh theory was accepted to explain Buoyancy-induced convective flows while the newly introduced theories explain the Benard convection. The Rayleigh theory and other theories modeled on it explain the conditions required for the onset of convective flow but fails to explain what happens once the flow is initiated.

STABILITY ANALYSIS

As Convection is motion due to force imbalance, it is often convenient to analyze it in terms of Stability. This analysis is given in the accompanying diagram.

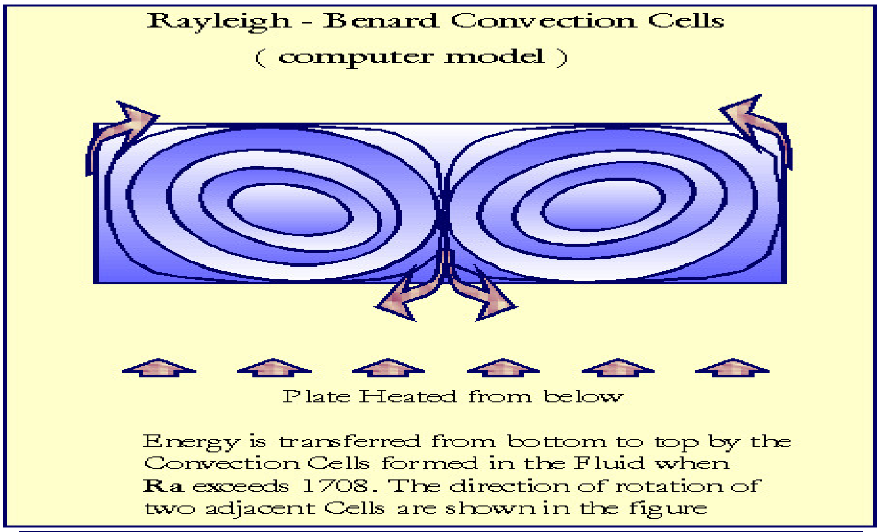


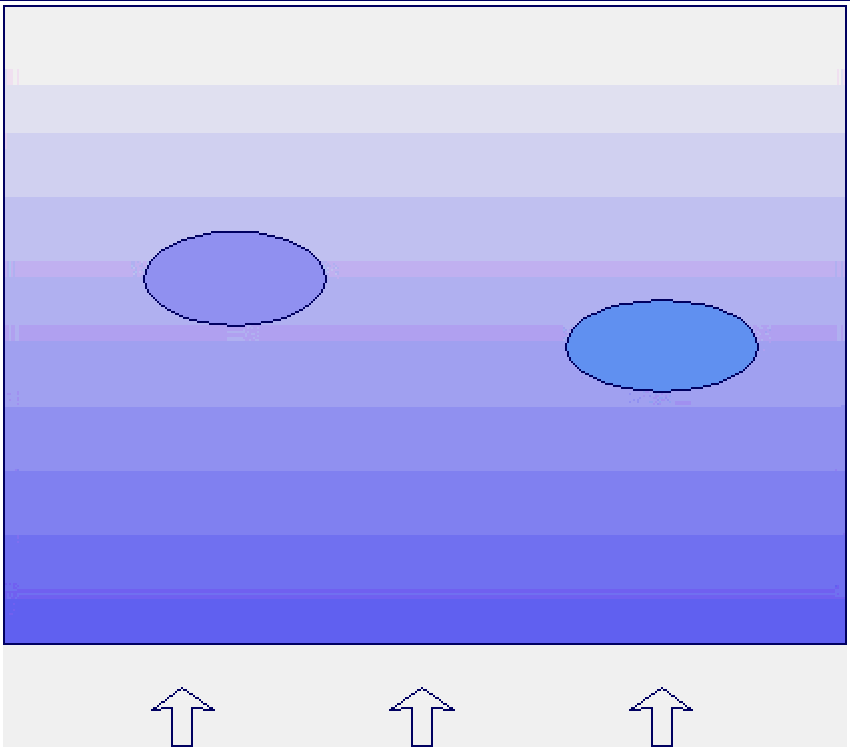
The system is usually found in the minimum energy state of a potential surface, which is what the lowest point in the bowl in the diagram represents. The **Rayleigh Number** by increasing converts this lowest point of maximum stability to the top and unique point of maximum instability of the system, as shown, resulting in the onset of Convective motion. By continuity we can predict that for a particular value of **Rayleigh Number** the potential surface will be a straight line of neutral stability. Experiments for the geometry considered reveal a *critical* **Rayleigh Number** of **1708** above which **convective instability** begins.

This Rayleigh theory of the onset of Convective motion in a fluid layer enclosed between two plates, though having many simplifying assumptions, nevertheless, explains the conditions required for the initiation of convection in real fluids successfully. But even the more comprehensive theories of later years cannot explain all the observed features of a fully developed convective flow in enclosed spaces. Only qualitative descriptions are possible.

CONVECTION CELLS

When the critical Rayleigh number is exceeded and as instability sets in, the hot layer tries to go up simultaneously when the cold upper layer tries to come down. Both things will not happen at the same time and the fluid avoids this stale mate by separating itself into a pattern of Convective Cells. In each cell the fluid rotates in a closed orbit and the direction of rotation alternates with successive cells. This roll when viewed in cross section resembles a bloated square the height of which is being determined by the width of the fluid layer. The following diagram shows the Cells in the two-dimensional view.

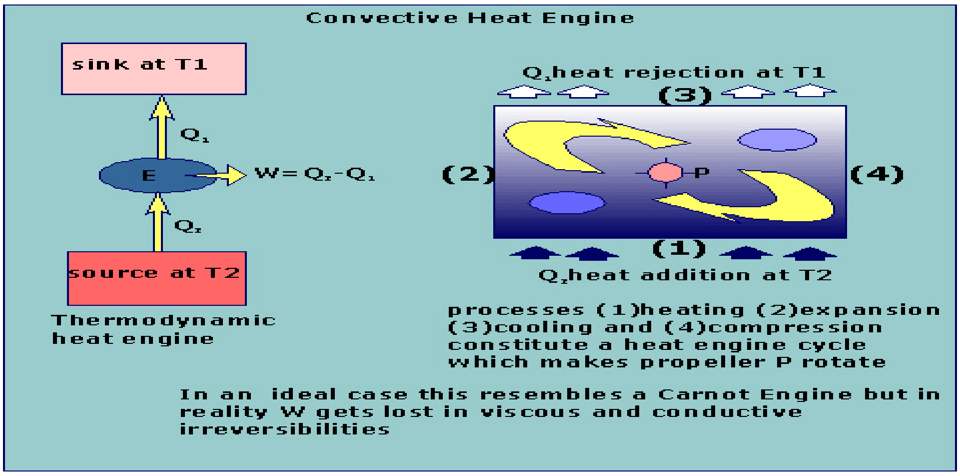




The above animation is just for visualization of what prompts the formation of a Convection Cell. It is by no means a Complete Description of the exact Fluid Mechanics and/or Heat Transfer inside a Cell !

THE CONVECTIVE ENGINE

A Free Convective flow is Thermodynamically a Heat Engine - a contrivance which by energy interaction with two external heat reservoirs gives useful work output.



1. Here the fluid packet considered to experience the force imbalance is composed of mass say dm. The actual movement of this packet is because it receives heat energy at the bottom from a heat source isothermally because of which it immediately expands. In effect it receives heat energy by undergoing an isothermal expansion process.
2. With this energy it increases its volume and as seen earlier experiences a force imbalance resulting in the displacement which causes further expansion as the packet raises. This results in an adiabatic expansion process in which the packet moving up can perform some work.
3. As it goes up the packet loses the remaining heat energy to the surrounding at the top to maintain equilibrium. This it does so as a isothermal compression process as it cools and contracts at the top.
4. Finally this cooled packet is what we get circulated back to the bottom, if we have to account for the `lost' mass in the bottom by mass conservation principle. This happens through an adiabatic compression process, as the packet further contracts as it comes down.

The fluid packet of mass dm executes a cycle comprising heating - expansion - cooling - compression. For anybody who remembers Thermodynamics it will immediately be evident that the processes executed by the packet here are the processes that constitute a **Carnot Heat Engine Cycle**.

CONCLUSION

The generality of the concept of convection is evident from the diverse examples stated in the beginning. Theories that exist today try to explain these diverse example which differ so largely in scale with very few related concepts and dimensionless numbers. Concepts like Wave Number and its relevance to Convection, the Landau Theory and its implications etc. have been left out of discussion in these pages. I will add some more, in the days to come. Meanwhile some references are added at the end for interested readers. In the years to come, it is expected that a complete general theory of Convective flow will be formulated to comprehend the still unexplored depths of this Phenomenon.

REFERENCES

1. M. G. Velarde & C. Normand - Convection, 1980, Scientific American
2. A. Bejan - ***Convection Heat Transfer***, 2nd ed.,1995, John Wiley & Sons
3. D. J. Tritton - ***Physical Fluid Dynamics***
4. Narasimhan, A., "Rayleigh-Benard Convection : Physics of a widespread phenomenon," ***Resonance***, **4(6)** pp 82-90, 1999.
5. Narasimhan, A., "Convective Carnot Engine: an easy approach to understanding convection;"

Physics Education, **35(3)** pp 178-181, 2000 .

6. A. V. Getling - ***Rayleigh Benard Convection : Structures and Dynamics***, 1998, World Scientific